

**The Fundamental Parameters of the Seidel Aberrations  
and  
Aplanatic Surfaces**

**Invited Talk for  
Tucson Student Chapter Optical Society of America**

**By**

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## Outline for TOSA Talk

- Seidel and His Aberrations
- Fundamental Parameters of the Seidel Aberrations
- Aplanatic Definitions
- Aplanatic Points & Conditions of a Surface
- What happens between these points/conditions
- Zemax Macro to Investigate
- Tools for optical design and aberrations reduction

# Philipp Ludwig von Seidel

- In 1856 L. Seidel (1821-1896) published the 1st complete mathematical basis, on the formation of an image by a lens
- Identified and named seven independent aberrations
- In general one degree of freedom to correct one aberration
- L. Seidel, Zur Dioptrik, Astronomische Nachrichten, No. 1027, 289 (1856); No. 1028, 305; No. 1029, 321

# Preparation Meets Opportunity

- After high school he spent one year of math tutoring under Schnurlein- student of Gauss
- 1840 Univ. of Berlin - student of Dirichlet, Encke
- 1842 Konigsberg - student of Bessel, Jacobi, Franz Neumann
- Studied Astronomy and Mathematical Analysis
- Problems with eyesight forced him into early retirement

# Seidel Aberrations

- Spherical Aberration
- Coma
- Astigmatism
- Field Curvature
- Distortion
- Axial Color
- Lateral Color

# Mathematical Formulas of Seidel Aberrations

<i>Spherical</i>	$W_{040} = -\frac{1}{8}S_I$	$S_I = -\sum A^2 y \Delta \left\{ \frac{u}{n} \right\}$	
<i>Coma</i>	$W_{131} = \frac{1}{2}S_{II}$	$S_{II} = -\sum A \bar{A} y \Delta \left\{ \frac{u}{n} \right\}$	
<i>Astigmatism</i>	$W_{222} = \frac{1}{2}S_{III}$	$S_{III} = -\sum \bar{A}^2 y \Delta \left\{ \frac{u}{n} \right\}$	
<i>Field Curvature</i>	$W_{220P} = \frac{1}{4}S_{IV}$	$S_{IV} = -\sum \aleph^2 P$	$\aleph = \text{Lagrange Invariant}$
<i>Distortion</i>	$W_{311} = \frac{1}{2}S_V$	$S_V = -\sum \frac{\bar{A}}{A} \left[ \aleph^2 P + \bar{A}^2 y \Delta \left\{ \frac{u}{n} \right\} \right]$	
<i>Axial Color</i>	$\delta_\lambda W_{020} = \frac{1}{2}C_L$	$C_L = \sum A y \Delta \left\{ \frac{\delta_n}{n} \right\}$	$\frac{\delta_n}{n} = \frac{n-1}{nV}$
<i>Lateral Color</i>	$\delta_\lambda W_{111} = \frac{1}{2}C_T$	$C_T = \sum \bar{A} y \Delta \left\{ \frac{\delta_n}{n} \right\}$	

*Alternate Form for Distortion if  $\bar{A} = 0$*

$$\Delta \left\{ \frac{u}{n} \right\} = A \Delta \left\{ \frac{1}{n^2} \right\} - yP \quad \aleph = \bar{A}y - A\bar{y}$$

$$S_V = -\sum \bar{A} \left[ \bar{A}^2 y \Delta \left\{ \frac{1}{n^2} \right\} - (\aleph - \bar{A}y) \bar{y}P \right]$$

## Fundamental Parameters of the Seidel Aberrations

### Refraction Invariant of the Marginal and Chief Rays

$$A \equiv ni = n'i' = nu + nyc = n'u' + n'yc$$

$$\bar{A} \equiv n\bar{i} = n'\bar{i}' = n\bar{u} + n\bar{y}c = n'\bar{u}' + n'\bar{y}c$$

$y$  is the marginal ray height at the surface in question

$$\Delta \left( \frac{u}{n} \right) = \left( \frac{u'}{n'} - \frac{u}{n} \right)$$

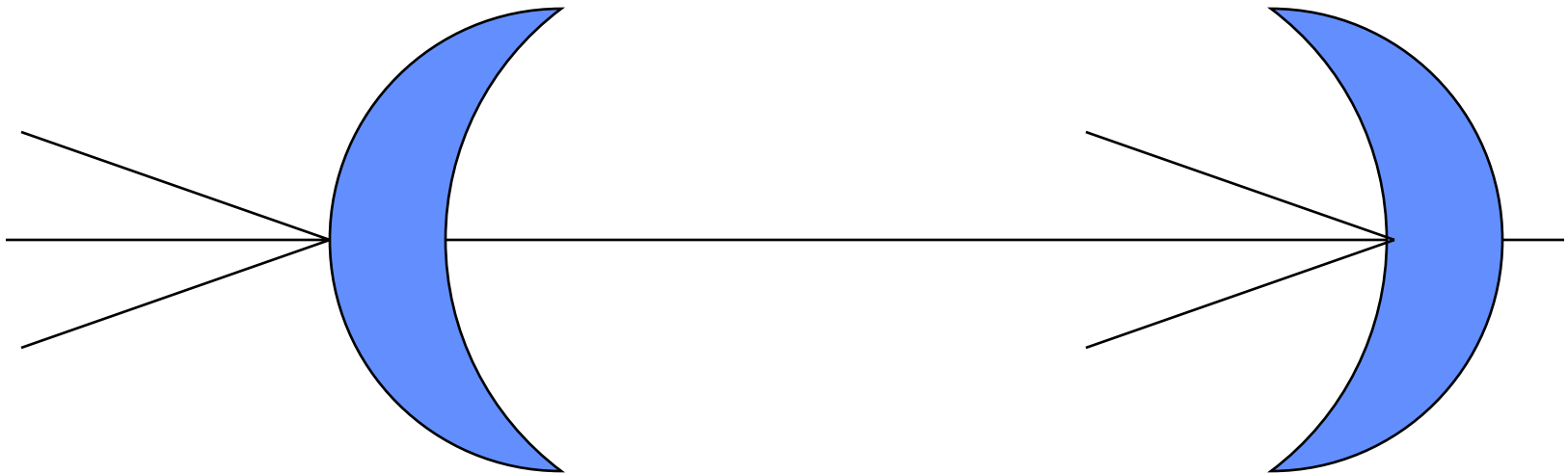
### **Aplanatic Surface: Zero Spherical and Coma at the Surface**

## Methods to make Aberrations Zero at a Surface

$$S_I = -\sum A^2 y \Delta \left( \frac{u}{n} \right) \quad S_{II} = -\sum A \bar{A} y \Delta \left( \frac{u}{n} \right) \quad S_{III} = -\sum \bar{A}^2 y \Delta \left( \frac{u}{n} \right)$$

- Aberration at surface is product of three or four quantities
- Make  $y = \text{zero}$
- Make refraction invariant(s) zero
- Make delta  $u/n$  zero

# Marginal Ray Height $y=0$

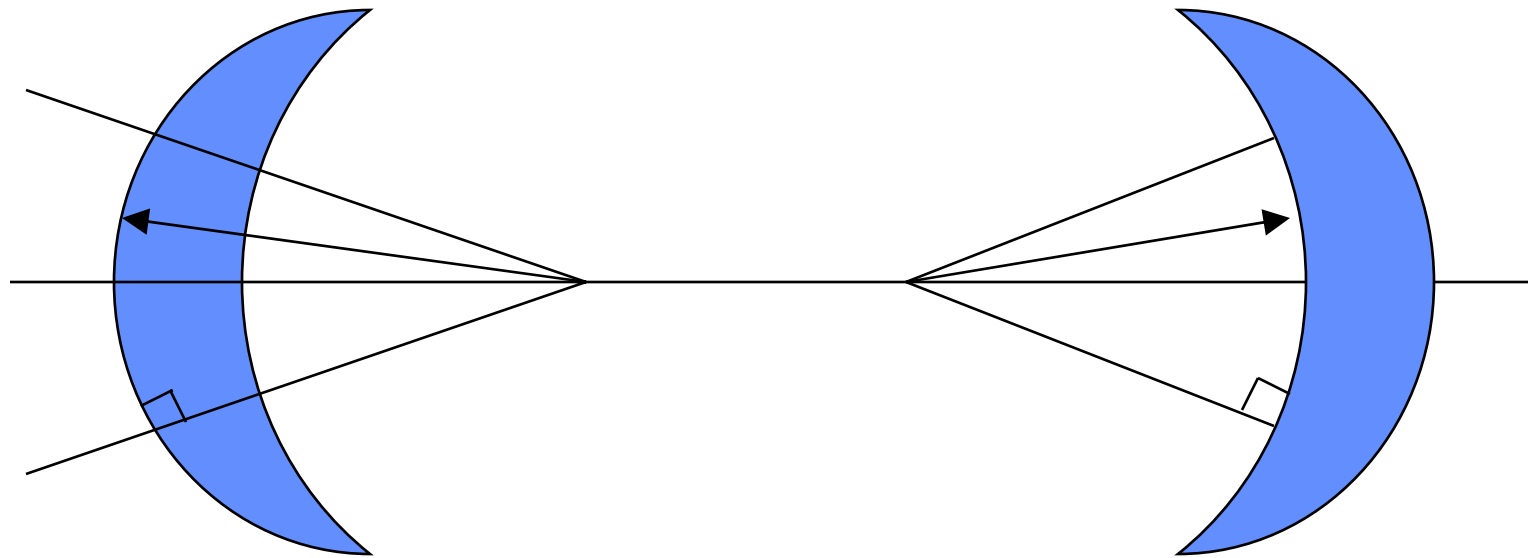


Convex Surface

Concave Surface



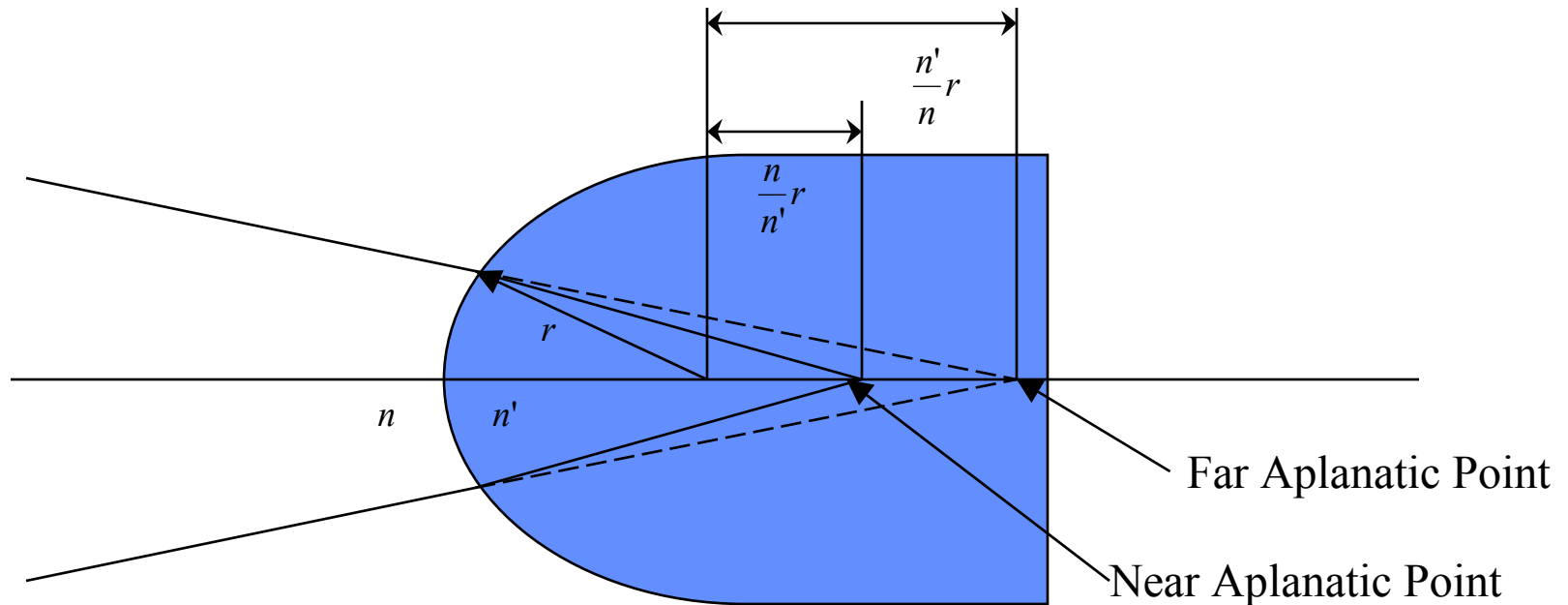
# Marginal Ray Refraction Invariant $A=0$ because $AOI=0$



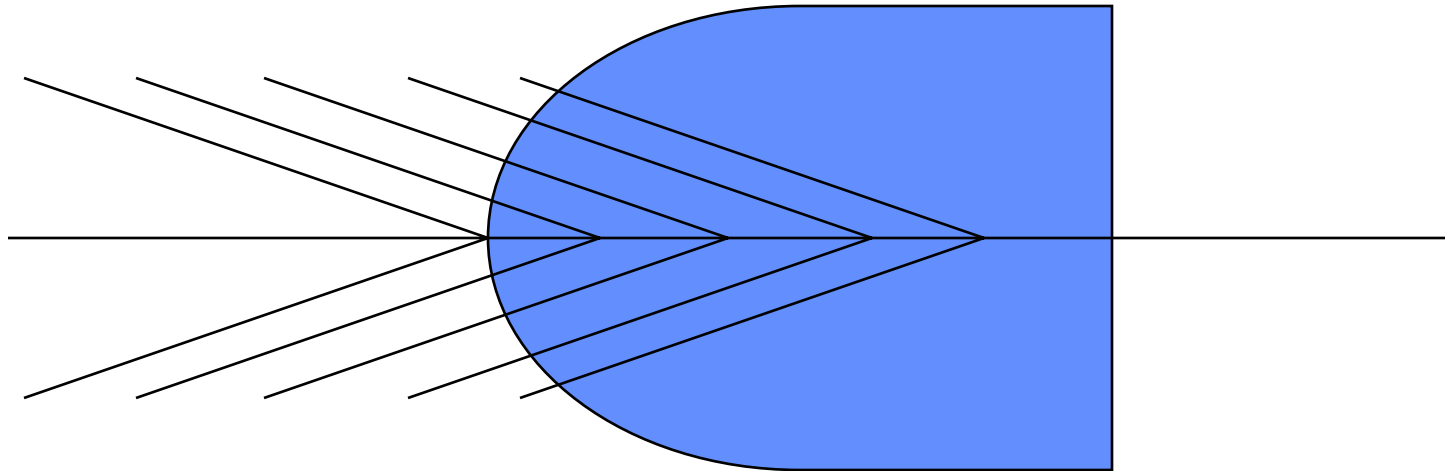
Convex Surface

Concave Surface

Delta  $u/n=0$  when aiming for far aplanatic point and refracts to near aplanatic point

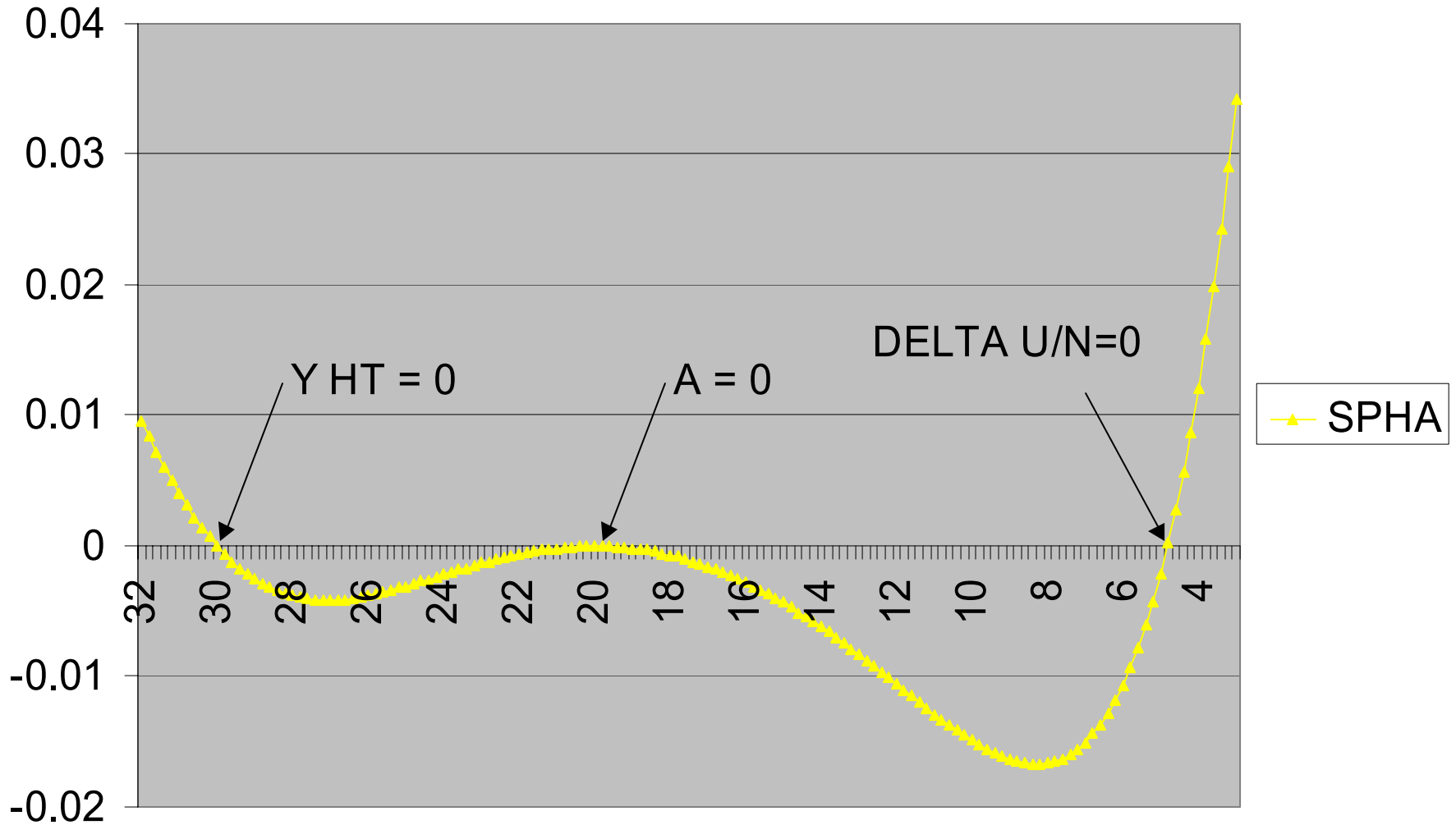


# Zooming through the aplanatic conditions of a spherical refractive surface w/ Zemax Macro

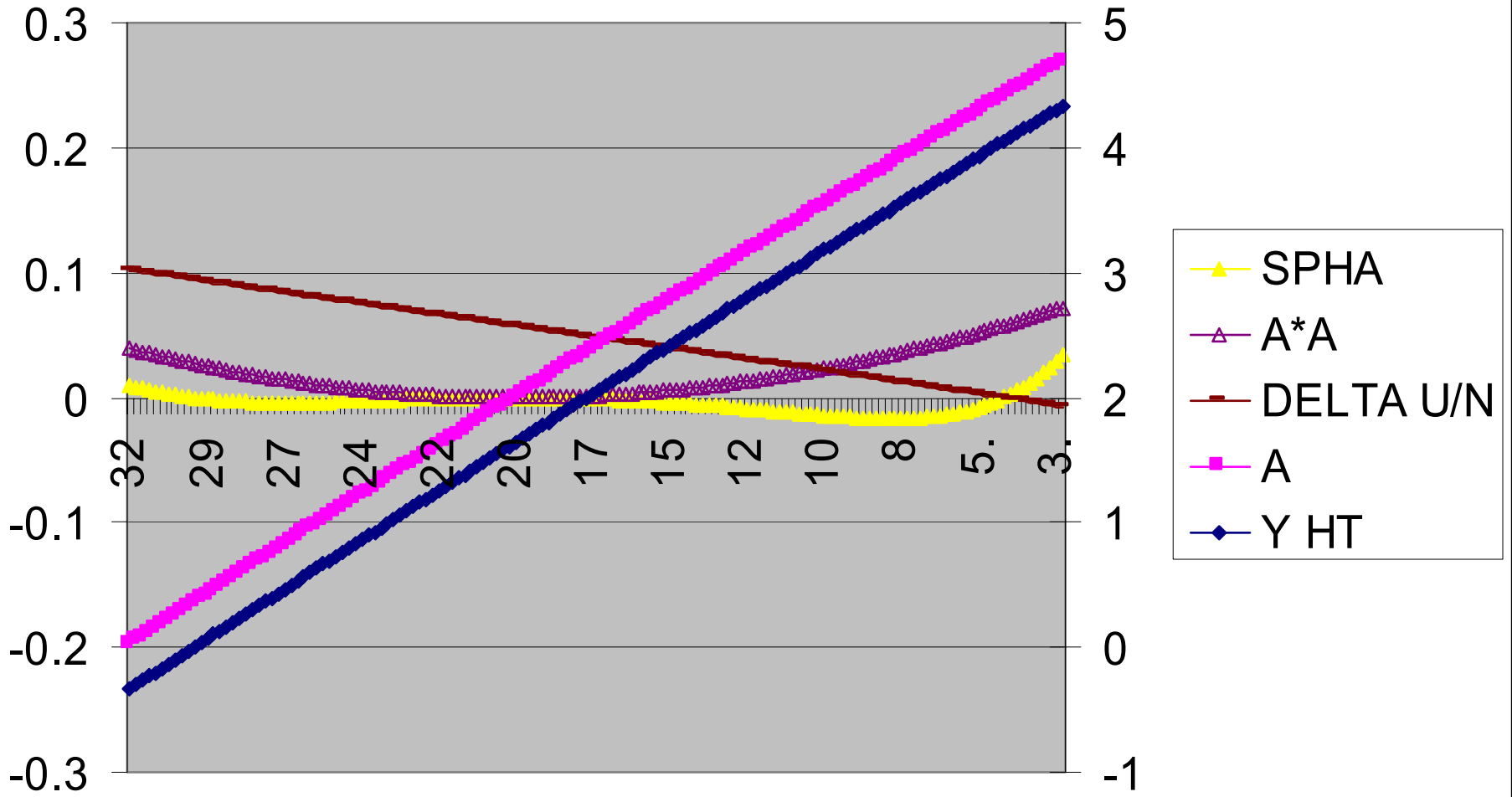


Please Note: Refraction of rays not shown

### Seidel Coefficient S1

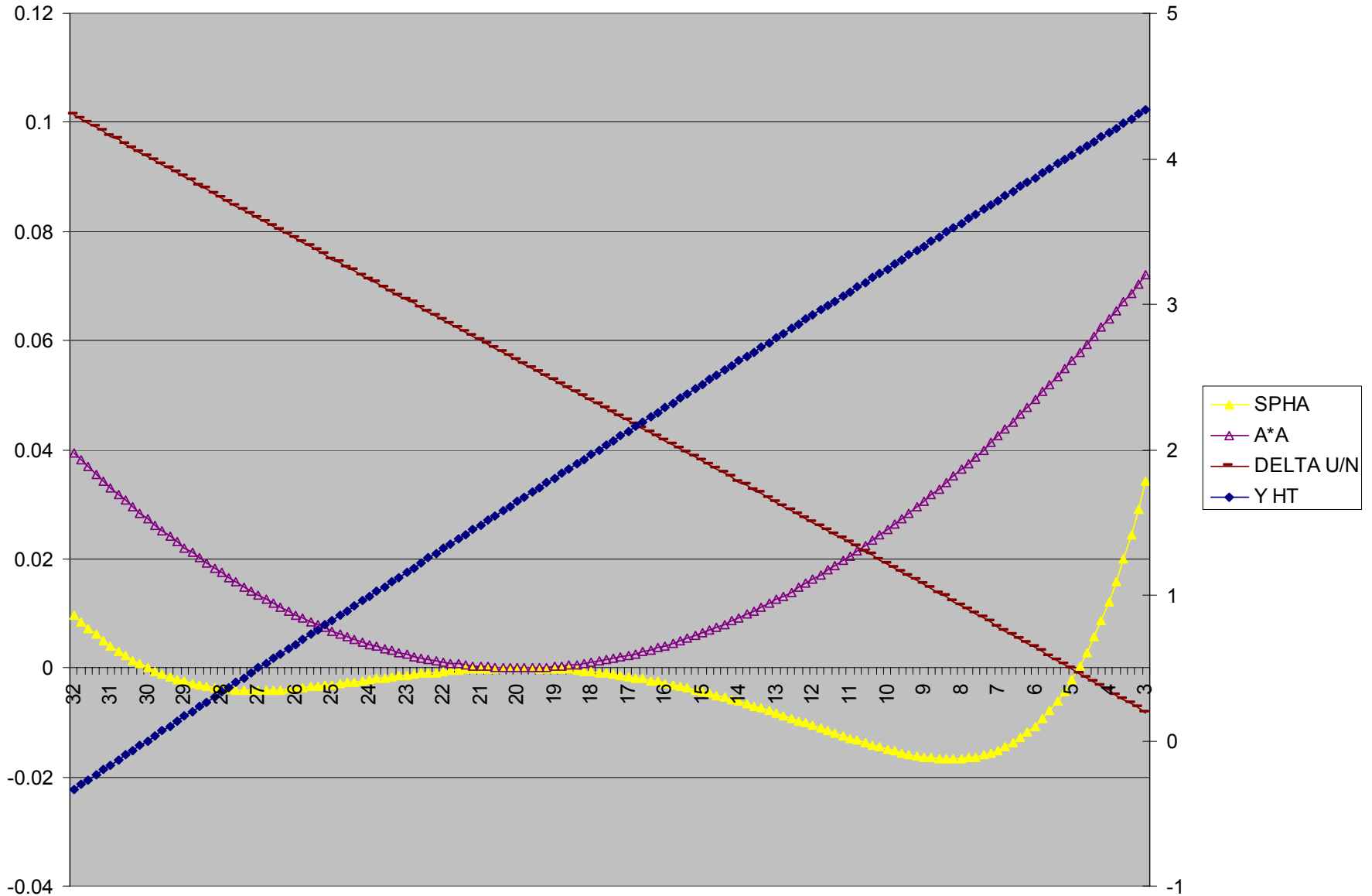


# Seidel Coefficient S1





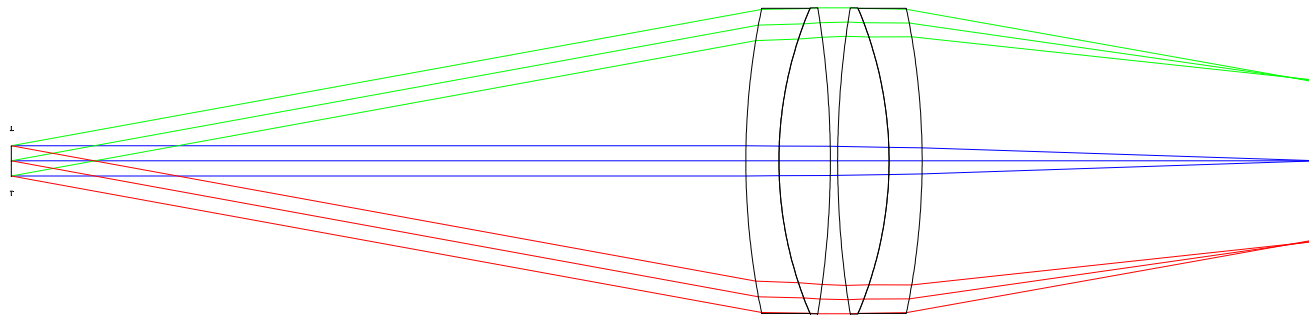
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# Plossl Eyepiece Design

Please Note: This optical design is backwards

SF12/Bk7 Both



	W040	W131	W222	W220P	W311	W020	W111
TOT	0.02	0.45	2.20	0.66	28.74	-0.29	-6.20

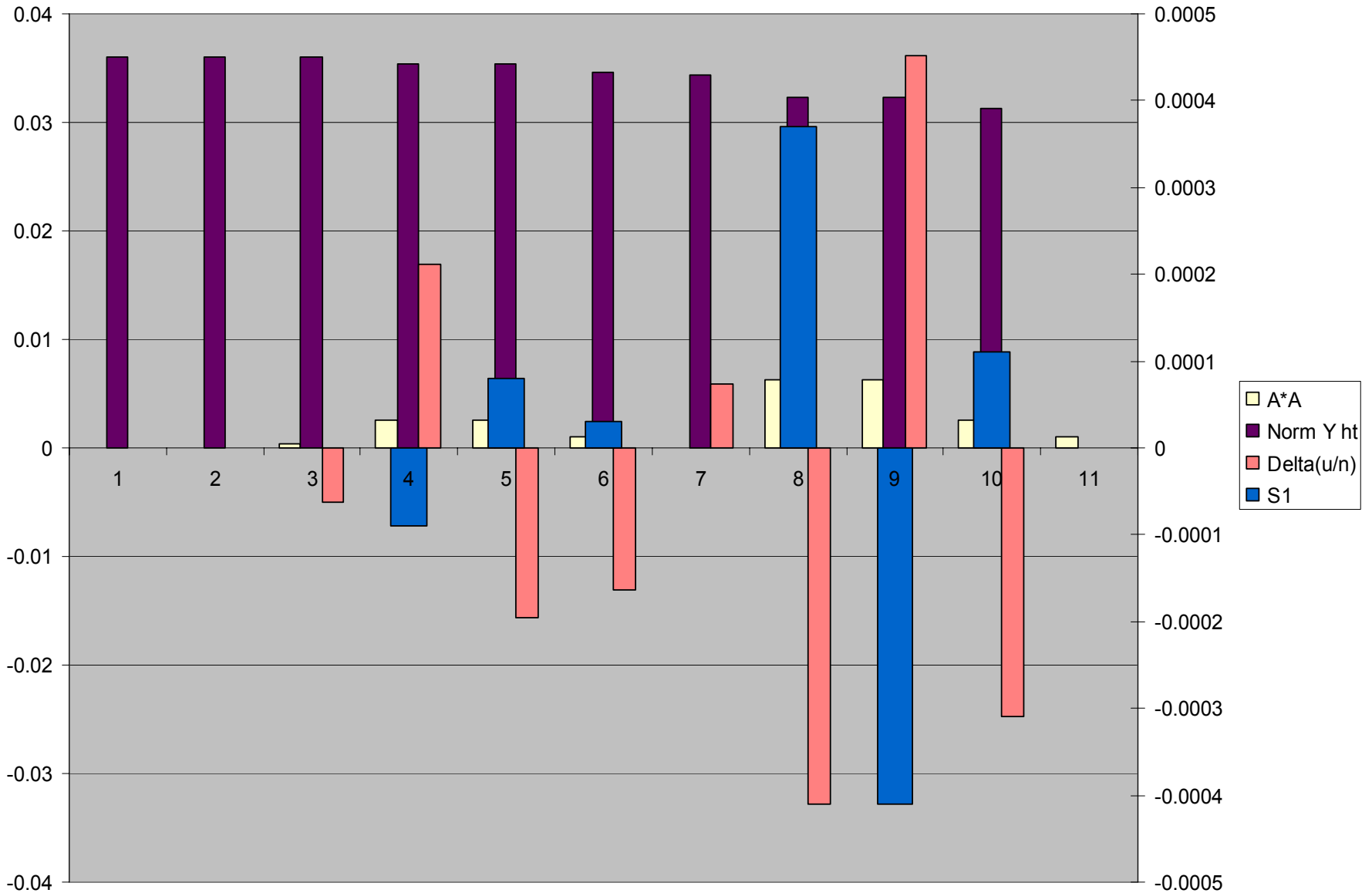
LAYOUT

PLOESSL EYEPIECE  
 THU JAN 4 2001  
 TOTAL LENGTH: 177.95883 MM

OPD ASSOCIATES  
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Plossl Eyepiece Seidel Coefficient S1 vs Surface #

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# Summary of Seidel's

- “A good lens designer must understand what is behind 3rd order aberrations and theory. It is crucial, critical, and essential that you know the 3rd order aberrations”
- Break the Seidel Aberrations down to their fundamental parameters
- Understand how optical design variables affect these fundamental parameters and you have arrived at the genesis of 3rd order lens design theory
- My approach to understanding aplanatic conditions
- Thanks you for your invitation and attention